Supplementary Materials: Online IMU Intrinsic Calibration: Is it Necessary?

Yulin Yang, Patrick Geneva, Xingxing Zuo and Guoquan Huang

I. System Linearization

The linearized state transition matrix can be written as:

$$\mathbf{\Phi}_{I(k+1,k)} = \begin{bmatrix} \mathbf{\Phi}_{11} & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{\Phi}_{14} & \mathbf{0}_3 \\ \mathbf{\Phi}_{21} & \mathbf{I}_3 & \mathbf{I}_3\Delta t_k & \mathbf{\Phi}_{24} & \mathbf{\Phi}_{25} \\ \mathbf{\Phi}_{31} & \mathbf{0}_3 & \mathbf{I}_3 & \mathbf{\Phi}_{34} & \mathbf{\Phi}_{35} \\ \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{I}_3 & \mathbf{0}_3 \\ \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{I}_3 \end{bmatrix} \quad (1)$$

$$\mathbf{\Phi}_{I,in} = \begin{bmatrix} \mathbf{\Phi}_{16} & \mathbf{0}_{3\times6} & \mathbf{0}_3 \\ \mathbf{\Phi}_{26} & \mathbf{\Phi}_{27} & \mathbf{\Phi}_{28} \\ \mathbf{\Phi}_{36} & \mathbf{\Phi}_{37} & \mathbf{\Phi}_{38} \\ \mathbf{0}_{3\times6} & \mathbf{0}_{3\times6} & \mathbf{0}_3 \\ \mathbf{0}_{3\times6} & \mathbf{0}_{3\times6} & \mathbf{0}_3 \end{bmatrix} \quad (2)$$

where:

$$\mathbf{\Phi}_{11} = {}^{I_{k+1}}_{I_k}\hat{\mathbf{R}},\ \mathbf{\Phi}_{14} = -\mathbf{J}_r(t_k)\Delta t_k\hat{\mathbf{D}}_w$$

$$\mathbf{\Phi}_{21} = -\lfloor {}^G\hat{\mathbf{p}}_{I_{k+1}} - {}^G\hat{\mathbf{p}}_{I_k} - {}^G\hat{\mathbf{v}}_{I_k}\Delta t_k + \frac{1}{2}{}^G\mathbf{g}\Delta t_k^2\rfloor {}^G_{I_k}\hat{\mathbf{R}}$$

$$\mathbf{\Phi}_{24} = \mathbf{H}^p_w\hat{\mathbf{D}}_w,\ \mathbf{\Phi}_{25} = -\mathbf{H}^p_a{}^I_a\hat{\mathbf{R}}\hat{\mathbf{D}}_a$$

$$\mathbf{\Phi}_{31} = -\lfloor {}^G\hat{\mathbf{v}}_{I_{k+1}} - {}^G\hat{\mathbf{v}}_{I_k} + {}^G\mathbf{g}\Delta t_k\rfloor {}^G_{I_k}\hat{\mathbf{R}}$$

$$\mathbf{\Phi}_{34} = \mathbf{H}^v_w\hat{\mathbf{D}}_w,\ \mathbf{\Phi}_{35} = -\mathbf{H}^v_a{}^I_a\hat{\mathbf{R}}\hat{\mathbf{D}}_a$$

$$\mathbf{\Phi}_{16} = \mathbf{J}_r(t_k)\Delta t_k\mathbf{H}_{Dw}$$

$$\mathbf{\Phi}_{26} = -\mathbf{H}^p_w\mathbf{H}_{Dw},\ \mathbf{\Phi}_{36} = -\mathbf{H}^v_w\mathbf{H}_{Dw}$$

$$\mathbf{\Phi}_{27} = \mathbf{H}^p_a{}^I_a\hat{\mathbf{R}}\mathbf{H}_{Da},\ \mathbf{\Phi}_{37} = \mathbf{H}^v_a{}^I_a\hat{\mathbf{R}}\mathbf{H}_{Da}$$

$$\mathbf{\Phi}_{28} = \mathbf{H}^p_a\lfloor {}^I\hat{\mathbf{a}}\rfloor,\ \mathbf{\Phi}_{38} = \mathbf{H}^v_a\lfloor {}^I\hat{\mathbf{a}}\rfloor$$

In order to simplify the derivation, we have made the following substitution:

$$\mathbf{H}^p_a = {}^G_{I_k}\hat{\mathbf{R}}\mathbf{\Xi}_2, \quad \mathbf{H}^p_w = {}^G_{I_k}\hat{\mathbf{R}}\mathbf{\Xi}_4 \quad (3)$$

$$\mathbf{H}^v_a = {}^G_{I_k}\hat{\mathbf{R}}\mathbf{\Xi}_1, \quad \mathbf{H}^v_w = {}^G_{I_k}\hat{\mathbf{R}}\mathbf{\Xi}_3 \quad (4)$$

where $\mathbf{\Xi}_i, i = 1\dots4$ can be evaluated with analytic expression in [2] or with numerical integration (e.g., RK4). The Jacobians for the noise term:

$$\mathbf{G}_k = \begin{bmatrix} -\mathbf{J}_r(t_k)\Delta t_k\hat{\mathbf{D}}_w & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 \\ \mathbf{H}^p_w\hat{\mathbf{D}}_w & -\mathbf{H}^p_a{}^I_a\hat{\mathbf{R}}\hat{\mathbf{D}}_a & \mathbf{0}_3 & \mathbf{0}_3 \\ \mathbf{H}^v_w\hat{\mathbf{D}}_w & -\mathbf{H}^v_a{}^I_a\hat{\mathbf{R}}\hat{\mathbf{D}}_a & \mathbf{0}_3 & \mathbf{0}_3 \\ \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{I}_3\Delta t_k & \mathbf{0}_3 \\ \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{I}_3\Delta t_k \\ \mathbf{0}_{15\times3} & \mathbf{0}_{15\times3} & \mathbf{0}_{15\times3} & \mathbf{0}_{15\times3} \end{bmatrix}$$

The covariance propagation for IMU and intrinsic states can be hence written as:

$$\mathbf{P}_{k+1} = \begin{bmatrix} \mathbf{\Phi}_I & \mathbf{\Phi}_{I,in} \\ \mathbf{0}_{15} & \mathbf{I}_{15} \end{bmatrix} \mathbf{P}_k \begin{bmatrix} \mathbf{\Phi}_I & \mathbf{\Phi}_{I,in} \\ \mathbf{0}_{15} & \mathbf{I}_{15} \end{bmatrix}^\top + \mathbf{G}_k\mathbf{Q}_{Id}\mathbf{G}_k^\top \quad (5)$$

where $\mathbf{P}$ denotes covariances for IMU and intrinsic states. $\mathbf{G}_{Id}$ represent the discrete covariance for IMU measurement noise.

II. Simulation Parameters

TABLE II shows the necessary parameters to reproduce the paper results. IMU Scale refers to the diagonal terms in $\mathbf{D}_a$ and $\mathbf{D}_w$. IMU Skew parameters refer to all the off diagonal terms in $\mathbf{D}_a$ and $\mathbf{D}_w$. $d_i, i = 1\dots4$ denote the 4 distortion parameters in camera model. All these parameters can be found and setup in OpenVINS [1].

III. Motion Comparison Between EuRoc MAV datasets and TUM VI datasets

Since the IMU calibration is based on a sliding-window filter, we compute the sampled standard deviations for local angular velocity and linear acceleration reading within around 0.5 seconds interval (approximate 100 IMU readings) and show the results in Fig. 1.

As clear from the figure, the underactuated MAV cannot fully excite the 3D motion of vehicles, hence the vehicle demonstrates a much smaller local angular velocity change (most cases no more than 6 degrees within an local window) than the handheld TUM datasets. We use the EuRoc MAV V101 and TUM VI Room1 as example. All the 6 axes of sensor in TUM VI Room1 dataset are fully excited, whereas the motions for Firefly in Eurco MAV are much smoother in the local time windows compared to TUM, especially for the angular velocity ($\boldsymbol{\omega}$). Hence, the vehicle's motions are close to the degenerate motions for IMU intrinsic calibration and degrade the system performance.

References

[1] Patrick Geneva, Kevin Eckenhoff, Woosik Lee, Yulin Yang, and Guoquan Huang. Openvins: A research platform for visual-inertial estimation. In *IROS 2019*

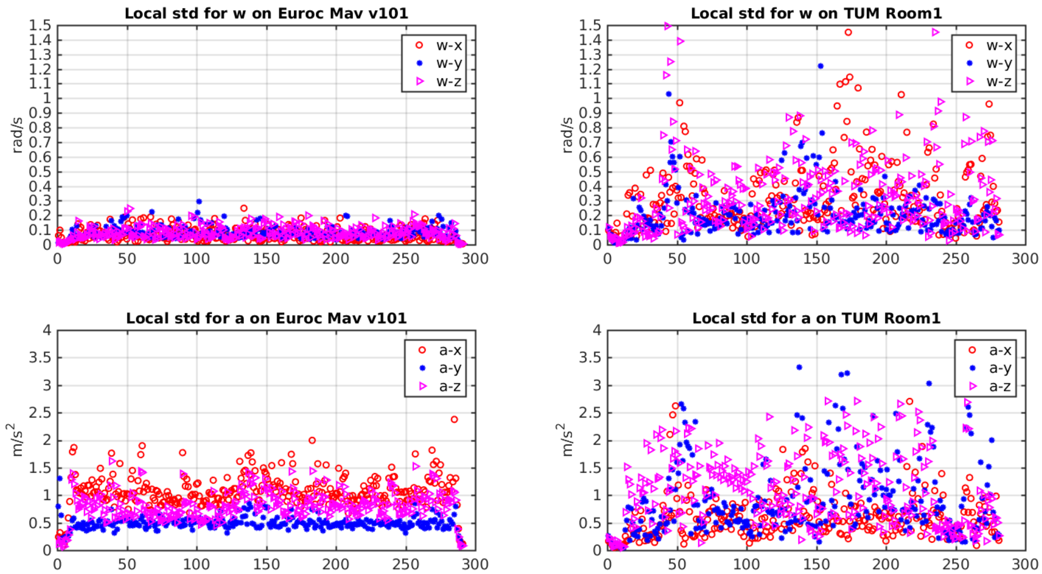

Fig. 1: The readings comparison between EuRoc MAV datasets (V101) and TUM VI datasets (Room1). We compute the sampled standard deviation for every 100 IMU readings (about 0.5s given 200Hz IMU). The larger the standard deviations are, the more dynamic the motions are.

Workshop on Visual-Inertial Navigation: Challenges and Applications, Macau, China, November 2019. URL https://github.com/rpng/open_vins.

[2] Yulin Yang, Benzun P. W. Babu, Chuchu Chen, Guoquan Huang, and Liu Ren. Analytic combined imu integrator (aci^2) with application to visual-inertial navigation. In *2020 IEEE International Conference on Robotics and Automation (ICRA)*, May 2020.