

Visual-Inertial-Wheel Odometry with Online Calibration

Woosik Lee, Kevin Eickenhoff, Yulin Yang, Patrick Geneva, and Guoquan Huang

Abstract—In this paper, we introduce a novel visual-inertial-wheel odometry (VIWO) system for ground vehicles, which efficiently fuses multi-modal visual, inertial and 2D wheel odometry measurements in a sliding-window filtering fashion. As multi-sensor fusion requires both intrinsic and extrinsic (spatiotemporal) calibration parameters which may vary over time during terrain navigation, we propose to perform VIWO along with online sensor calibration of wheel encoders' intrinsic and extrinsic parameters. To this end, we analytically derive the 2D wheel odometry measurement model from the raw wheel encoders' readings and optimally fuse this 2D relative motion information with 3D visual-inertial measurements. Additionally, an observability analysis is performed for the linearized VIWO system, which identifies five commonly-seen degenerate motions for wheel calibration parameters. The proposed system has been validated extensively in both Monte-Carlo simulations and real-world experiments in large-scale urban driving scenarios.

I. INTRODUCTION

Autonomous ground vehicles have found their ways into many applications from autonomous driving and warehousing to military and agriculture robots navigating off-road rough terrains. High-precision consistent 3D localization with low-cost multi-modal sensors (given that engineered robotic systems have limited cost budgets) is essential to enable such autonomy. Among all possible sensor suites, cameras, IMUs and wheel encoders are appealing because of their ubiquity and low costs while providing sufficient information for 3D motion estimation. While visual-inertial odometry (VIO) has witnessed great success in recent years and has shown that even a single camera and IMU can provide accurate 3D motion tracking [1], VIO can suffer from unobservability if it undergoes planar motions such as constant acceleration which is a common case for ground vehicles. Thus, it is necessary to further aid VIO with additional sensors such as wheel encoders (which are typically available on wheeled ground vehicles) [2], [3]. As such, in this work we aim to develop an efficient visual-inertial-wheel odometry (VIWO) algorithm for ground vehicles.

It is well understood that accurate calibration is a prerequisite for multi-sensor fusion, which is often obtained offline and assumed to be constant during operation (e.g., [2]). However, calibration parameters may change over time, for example, due to mechanical vibration and environmental effects, which, if not addressed properly, can hurt the estimation performance (e.g., see Fig. 1). We thus advocate

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The authors are with the Robot Perception and Navigation Group (RPNG), University of Delaware, Newark, DE 19716, USA. {woosik, keck, yuyang, pgeneva, ghuang}@udel.edu

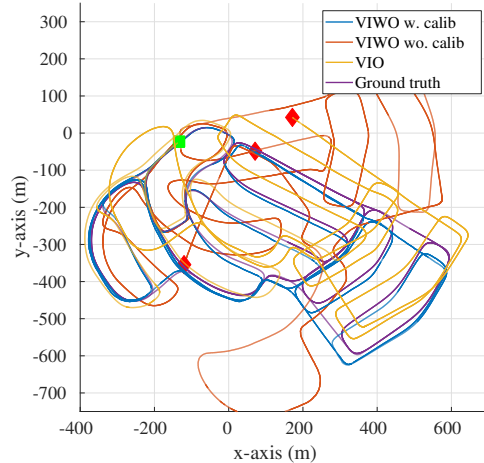


Fig. 1: Simulation results for VIWO with online calibration (blue), VIWO without calibration (red), and VIO (yellow). The green square and red diamond correspond to the start and end of the 8.9km trajectory, respectively. to perform online sensor calibration of both intrinsic and extrinsic parameters in the proposed VIWO.

In particular, an efficient, consistent, and tightly-coupled VIWO estimator is developed, which also performs online sensor calibration of both the differential-drive model's [4] intrinsics and the spatiotemporal extrinsics between the wheel odometer and IMU/camera. Note that IMU/camera calibration has been well studied in our prior work [5] and is assumed known in this work without a loss of generality. Specifically, the main contributions of this work include:

- We develop an efficient MSCKF-based VIWO estimator to optimally fuse IMU, camera, and preintegrated wheel measurements, which particularly models the wheel-IMU time offset and performs online calibration of both intrinsic and extrinsic spatiotemporal parameters.
- We derive the linearized state transition matrix by implicitly incorporating the MSCKF's stochastic cloning, and perform observability analysis to find degenerate motions that cause calibration parameters unidentifiable.
- We evaluate the proposed VIWO extensively in realistic simulations, showing calibration convergence, robustness to poor initial calibration values, and estimator consistency, as well as the identified degenerate motions. Moreover, we validate the proposed VIWO system in a large-scale urban driving experiment.

II. RELATED WORK

Rich literature exists on differential drive kinematic models and their offline intrinsic calibration [6]–[9], online calibration within a graph optimization framework [10], and the filtering framework [11], [12]. These methods focus on processing odometry measurements at the sensor rate and thus can suffer computational burdens as the frequency increases.

In contrast, preintegration of odometry measurements over time creates an inferred measurement about the change of the robot state and compresses high frequency information over a time period, thus reducing computational burdens. For example, wheel odometry measurements are preintegrated for 2D localization with online intrinsic calibration in [13], [14]. Zuo et al. [3] presented a 3D sliding-window optimization of an instantaneous center of rotation (ICR)-based for skid-steering robots [15], along with online wheel intrinsic calibration. Although the ICR model encompasses differential drive models as a special case, the ICR intrinsic parameters were modeled as random walk, while the differential drive model assumes mechanically fixed intrinsic parameters and thus provides additional information.

Offline extrinsic calibration of the wheel odometry and other sensors has also been well studied. For example, Antonelli et al. [16] performed relative 3D orientation and 2D position calibration between the camera and the odometry frame with known landmarks, while Heng et al. [17] calibrated the wheel and multi-camera without the need of environmental targets. Online calibration approaches also exist, for instance, in [18]–[21] graph optimization was used for 6DOF spatial calibration. However, all these methods assumed the sensors are time-synchronized. As different non-synchronized sensors provide measurements in different clocks, it is required to know the time offset between sensor clocks in order to fuse measurements with a common timeline. As shown in our previous work [22], even a small time offset error between the sensors can greatly affect estimation errors. To the best of our knowledge, no work has yet investigated this time offset calibration between wheel odometry and IMU, although significant research efforts in VIO has studied both online calibration of IMU/camera time offset [23]–[27] and offline calibration [28]–[31].

Observability analysis [32], [33] is of practical importance, for example, due to its ability to identify degenerate motions which can affect estimation accuracy [2], [34], and design consistent estimators [35]. Understanding observability properties and degenerate motions is particularly important for ground vehicles as their constrained motions often prevent full excitation of the sensor platform. For the differential drive model, Martinelli et al. [36] showed that with camera and wheel sensors, the wheel odometry intrinsic and 2D extrinsic calibration parameters are observable up to the scale under general motion. The author also found a degenerate motion where parameters are unobservable when each wheel keeps the same speed ratio. Censi et al. [37] performed intrinsic and 2D extrinsic calibration between wheel and laser sensors and showed three motion profiles for which the parameters are observable, but did not provide specific degenerate motions. Zuo et al. [3] analyzed the observability and degenerate motions for wheel intrinsics of a skid-steering model, while Yang et al. [5] did a comprehensive observability and degenerate motion analysis on extrinsic and time offset parameters for a IMU/camera pair.

III. MSCKF-BASED VIO

In this section, we briefly review the standard MSCKF framework [38] from which the proposed VIWO is built on.

Specifically, at time t_k , the state vector \mathbf{x}_k consists of the current inertial state \mathbf{x}_{I_k} and n historical IMU pose clones \mathbf{x}_{C_k} represented in the global frame $\{G\}$:

$$\mathbf{x}_k = [\mathbf{x}_{I_k}^\top \quad \mathbf{x}_{C_k}^\top]^\top \quad (1)$$

$$\mathbf{x}_{I_k} = [{}^G \bar{q}^\top \quad {}^G \mathbf{p}_{I_k}^\top \quad {}^G \mathbf{v}_{I_k}^\top \quad \mathbf{b}_g^\top \quad \mathbf{b}_a^\top]^\top \quad (2)$$

$$\mathbf{x}_{C_k} = [{}^{I_{k-1}} \bar{q}^\top \quad {}^G \mathbf{p}_{I_{k-1}}^\top \quad \dots \quad {}^{I_{k-n}} \bar{q}^\top \quad {}^G \mathbf{p}_{I_{k-n}}^\top]^\top \quad (3)$$

where ${}^I_k \bar{q}$ is the JPL unit quaternion [39] corresponding to the rotation ${}^I_k \mathbf{R}$ from $\{G\}$ to IMU frame $\{I\}$, ${}^G \mathbf{p}_{I_k}$ and ${}^G \mathbf{v}_{I_k}$ are the position and velocity of $\{I\}$ in $\{G\}$, and \mathbf{b}_g and \mathbf{b}_a are the biases of the gyroscope and accelerometer. We define $\mathbf{x} = \hat{\mathbf{x}} \boxplus \tilde{\mathbf{x}}$, where \mathbf{x} is the true state, $\hat{\mathbf{x}}$ is its estimate, $\tilde{\mathbf{x}}$ is the error state, and the operation \boxplus which maps the error state vector to its corresponding manifold [40].

A. IMU Kinematic Model

The state is propagated forward in time using the IMU linear acceleration \mathbf{a}_m and angular velocity $\boldsymbol{\omega}_m$ measurements:

$$\mathbf{a}_m = \mathbf{a} + {}^I_G \mathbf{R} \mathbf{g} + \mathbf{b}_a + \mathbf{n}_a, \quad \boldsymbol{\omega}_m = \boldsymbol{\omega} + \mathbf{b}_g + \mathbf{n}_g \quad (4)$$

where \mathbf{a} and $\boldsymbol{\omega}$ are the true local acceleration and angular velocity, $\mathbf{g} \approx [0 \ 0 \ 9.81]^\top$ is the global gravity, and \mathbf{n}_a and \mathbf{n}_g are zero mean Gaussian noises. We propagate the state estimate and covariance from time t_k to t_{k+1} based on the standard inertial kinematic model $\mathbf{f}(\cdot)$ [39] under the assumption of zero noise:

$$\hat{\mathbf{x}}_{k+1|k} = \mathbf{f}(\hat{\mathbf{x}}_{k|k}, \mathbf{a}_m, \boldsymbol{\omega}_m, \mathbf{0}, \mathbf{0}) \quad (5)$$

$$\mathbf{P}_{k+1|k} = \Phi(t_{k+1}, t_k) \mathbf{P}_{k|k} \Phi(t_{k+1}, t_k)^\top + \mathbf{Q}_k \quad (6)$$

where $\hat{\mathbf{x}}_{a|b}$ denotes the estimate at time t_a formed by processing the measurements up to time t_b , and Φ and \mathbf{Q} are the state transition matrix and discrete noise covariance.

B. Camera Measurement Model

Sparse corner features are detected and tracked over a window of images associated with the cloned frames \mathbf{x}_{C_k} . The resulting bearing measurements, \mathbf{z}_k , are given by:

$$\mathbf{z}_k = \mathbf{\Pi}({}^{C_k} \mathbf{p}_f) + \mathbf{n}_k \quad (7)$$

$${}^{C_k} \mathbf{p}_f = {}^C_I \mathbf{R}_G {}^I_k \mathbf{R} ({}^G \mathbf{p}_f - {}^G \mathbf{p}_{I_k}) + {}^C \mathbf{p}_I \quad (8)$$

where $\mathbf{\Pi}([x \ y \ z]^\top) = [\frac{x}{z} \quad \frac{y}{z}]^\top$ is the perspective projection, ${}^G \mathbf{p}_f$ is the 3D point feature, and $\{{}^C_I \mathbf{R}, {}^C \mathbf{p}_I\}$ are the camera to IMU extrinsic transformation.¹ Stacking all measurements corresponding to a single feature and performing linear marginalization of the feature's position (via a nullspace operation) results in a residual [38]:

$$\tilde{\mathbf{z}}_{c_k} = \mathbf{H}_{x_k} \tilde{\mathbf{x}}_k + \mathbf{n}_{f_k} \quad (9)$$

This then can be directly used in EKF update without storing features in the state, leading to substantial computational savings and bounding the state size.

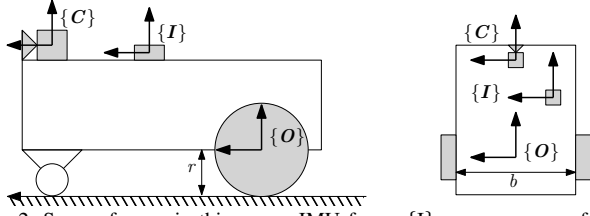


Fig. 2: Sensor frames in this paper: IMU frame $\{I\}$, camera sensor frame $\{C\}$, and odometry frame $\{O\}$ located at the center of the baselink

IV. WHEEL-ENCODER MEASUREMENT MODEL

Building upon the preceding VIO models, we now generalize our 3D motion tracking system to optimally incorporate 2D wheel-encoder measurements that are commonplace in ground vehicles. In particular, a ground vehicle is often driven by two differential (left and right) wheels mounted on a common axis (baselink), each equipped with an encoder providing local angular rate readings [4]:

$$\omega_{ml} = \omega_l + n_{\omega_l}, \quad \omega_{mr} = \omega_r + n_{\omega_r} \quad (10)$$

where ω_l and ω_r are the true angular velocities of each wheel, and n_{ω_l} and n_{ω_r} are the corresponding zero-mean white Gaussian noises. These encoder readings can be combined to provide 2D linear and angular velocities about the vehicle body or odometer frame $\{O\}$ at the center of the baselink:

$${}^O\omega = (\omega_r r_r - \omega_l r_l)/b, \quad {}^Ov = (\omega_r r_r + \omega_l r_l)/2 \quad (11)$$

where $\mathbf{x}_{WI} := [r_l \ r_r \ b]^\top$ are the left and right wheel radii and the baselink length, respectively.

A. Wheel Odometry Preintegration

As the wheel encoders typically provide measurements of higher rate (e.g., 100-500 Hz) than the camera, it would be too expensive to perform EKF update at their rate. To address this issue, in the proposed VIWO, we first integrate a set of wheel-encoder's velocity measurements over a time interval corresponding to the most recent two images and create a 2D relative-pose measurement of lower rate (i.e., preintegration). This preintegrated odometry measurement constrains the latest two camera poses that are already maintained in the MSCKF's clones. As a result, this process incurs only small extra computational overhead as the VIWO state vector remains the same (up to online calibration) as that of the standard VIO [see (1)].

Specifically, consider preintegrating a series of wheel-encoder measurements between two clone times t_k and t_{k+1} . The continuous-time 2D kinematic model for $t_\tau \in [t_k, t_{k+1}]$ is given by:

$${}^{O_k}\dot{\theta} = {}^{O_k}\omega \quad (12)$$

$${}^{O_k}\dot{x}_{O_\tau} = {}^{O_k}v \cos({}^{O_k}\theta) \quad (13)$$

$${}^{O_k}\dot{y}_{O_\tau} = {}^{O_k}v \sin({}^{O_k}\theta) \quad (14)$$

where ${}^{O_k}\theta$ is the local yaw angle, ${}^{O_k}x_{O_\tau}$ and ${}^{O_k}y_{O_\tau}$ are the 2D position of $\{O_\tau\}$ in the starting integration frame $\{O_k\}$.

¹In this paper we assume that the camera-IMU extrinsics along with the temporal calibration (i.e., the time offset between the two sensors), are known – which however can be calibrated either offline or online [41] – so that we can focus on the online intrinsic and extrinsic calibrations of the odometer and IMU in the proposed VIWO system.

Note that this model reveals the fact that the 2D orientation evolves over the integration period. We then integrate these differential equations from t_k to t_{k+1} and obtain the 2D relative pose measurement as follows:

$$\mathbf{z}_{k+1} = \begin{bmatrix} {}^{O_{k+1}}\theta \\ {}^{O_k}x_{O_{k+1}} \\ {}^{O_k}d_{O_{k+1}} \end{bmatrix} = \begin{bmatrix} \int_{t_k}^{t_{k+1}} {}^{O_t}\omega dt \\ \int_{t_k}^{t_{k+1}} {}^{O_t}v \cos({}^{O_t}\theta) dt \\ \int_{t_k}^{t_{k+1}} {}^{O_t}v \sin({}^{O_t}\theta) dt \end{bmatrix} \quad (15)$$

$$=: \mathbf{g}(\omega_{l(k:k+1)}, \omega_{r(k:k+1)}, \mathbf{x}_{WI}) \quad (16)$$

where $\omega_{(k:k+1)}$ denote all the wheel measurements integrated t_k to t_{k+1} . If both extrinsic and time offset (spatiotemporal) calibration parameters between the odometer and IMU/camera are perfectly known, the above integrated odometry measurements can be readily used in the MSCKF update as in [2]. However, in practice, this often is not the case, for example, due to inaccurate prior calibration or mechanical vibration. To cope with possible time-varying calibration parameters during terrain navigation, the proposed VIWO performs online calibration of the wheel-encoders' intrinsics \mathbf{x}_{WI} , the spatial extrinsics $\mathbf{x}_{WE} = [{}^O\bar{\mathbf{q}}^\top \ {}^O\mathbf{p}_I^\top]^\top$, and time offset ${}^O t_I$ between the odometer and IMU. Note again that the IMU and camera are assumed to be calibrated and synchronized for presentation brevity. To this end, we augment the state vector (1) with these parameters:

$$\mathbf{x}_k = [\mathbf{x}_{I_k}^\top \ \mathbf{x}_{C_k}^\top \ \mathbf{x}_{WE}^\top \ {}^O t_I \ \mathbf{x}_{WI}^\top]^\top \quad (17)$$

In what follows, we will derive in detail the relation between the preintegrated wheel odometry measurements (16) and the augmented state (17) by properly taking into account the intrinsic/extrinsic calibration parameters:

$$\mathbf{z}_{k+1} = \mathbf{h}(\mathbf{x}_{I_{k+1}}, \mathbf{x}_{C_{k+1}}, \mathbf{x}_{WE}, {}^O t_I, \mathbf{x}_{WI}) \quad (18)$$

B. Odometry Measurement wrt. Intrinsics

As evident from (11) and (16), the wheel-odometry integration entangles the intrinsics \mathbf{x}_{WI} . To address this issue, we linearize the preintegrated odometry measurements about the current estimate of the intrinsics while properly taking into account the measurement uncertainty due to the linearization errors of the intrinsics and the noise [see (16)]:

$$\mathbf{z}_{k+1} \simeq \mathbf{g}(\omega_{ml(k:k+1)}, \omega_{mr(k:k+1)}, \hat{\mathbf{x}}_{WI}) + \frac{\partial \mathbf{g}}{\partial \mathbf{x}_{WI}} \tilde{\mathbf{x}}_{WI} + \frac{\partial \mathbf{g}}{\partial \mathbf{n}_w} \mathbf{n}_w \quad (19)$$

where \mathbf{n}_w is the stacked noise vector whose τ -th block is corresponding to the encoder measurement noise at $t_\tau \in [t_k, t_{k+1}]$ (i.e., $[n_{\omega_{l,\tau}} \ n_{\omega_{r,\tau}}]^\top$) [see (10)].

To perform an EKF update with this measurement, the Jacobians with respect to both the intrinsics and the noise in (19) are required. As we incrementally compute the preintegration with the encoders' measurements available in the interval $[t_k, t_{k+1}]$, we can accordingly calculate the measurement Jacobians incrementally one step at a time. As compared to stacking a high-dimension noise Jacobian and \mathbf{n}_w , we incrementally build the final measurement noise covariance. Specifically, linearization of (15) at time t_τ yields the following recursive equations:

$${}^{O_{k+1}}\tilde{\theta} = {}^{O_k}\tilde{\theta} + \mathbf{H}_{1,\tau} \tilde{\mathbf{x}}_{WI} + \mathbf{H}_{2,\tau} \mathbf{n}_{w,\tau} \quad (20)$$

$${}^{O_k}\tilde{x}_{O_{\tau+1}} = {}^{O_k}\tilde{x}_{O_{\tau}} + \mathbf{H}_{3,\tau} {}^{O_{\tau}}\tilde{\theta} + \mathbf{H}_{4,\tau}\tilde{\mathbf{x}}_{WI} + \mathbf{H}_{5,\tau}\mathbf{n}_{\omega,\tau} \quad (21)$$

$${}^{O_k}\tilde{y}_{O_{\tau+1}} = {}^{O_k}\tilde{y}_{O_{\tau}} + \mathbf{H}_{6,\tau} {}^{O_{\tau}}\tilde{\theta} + \mathbf{H}_{7,\tau}\tilde{\mathbf{x}}_{WI} + \mathbf{H}_{8,\tau}\mathbf{n}_{\omega,\tau} \quad (22)$$

where the intermediate Jacobians, $\mathbf{H}_{l,\tau}, l = 1 \dots 8$, can be found in our companion technical report [42]. With the above equations, we can recursively compute the noise covariance $\mathbf{P}_{m,\tau+1}$ and the Jacobian $\frac{\partial \mathbf{g}_{\tau+1}}{\partial \tilde{\mathbf{x}}_{WI}}$ as follows:

$$\Phi_{tr,\tau} = \begin{bmatrix} 1 & 0 & 0 \\ \mathbf{H}_{3,\tau} & 1 & 0 \\ \mathbf{H}_{6,\tau} & 0 & 1 \end{bmatrix}, \Phi_{WI,\tau} = \begin{bmatrix} \mathbf{H}_{1,\tau} \\ \mathbf{H}_{4,\tau} \\ \mathbf{H}_{7,\tau} \end{bmatrix}, \Phi_{n,\tau} = \begin{bmatrix} \mathbf{H}_{2,\tau} \\ \mathbf{H}_{5,\tau} \\ \mathbf{H}_{8,\tau} \end{bmatrix} \quad (23)$$

$$\mathbf{P}_{m,\tau+1} = \Phi_{tr,\tau} \mathbf{P}_{m,\tau} \Phi_{tr,\tau}^{\top} + \Phi_{n,\tau} \mathbf{Q}_{\tau} \Phi_{n,\tau}^{\top} \quad (24)$$

$$\frac{\partial \mathbf{g}_{\tau+1}}{\partial \tilde{\mathbf{x}}_{WI}} = \Phi_{tr,\tau} \frac{\partial \mathbf{g}_{\tau}}{\partial \tilde{\mathbf{x}}_{WI}} + \Phi_{WI,\tau} \quad (25)$$

where \mathbf{Q}_{τ} is the noise covariance of wheel encoder measurement at t_{τ} . We recursively compute measurement noise covariance \mathbf{P}_m and the Jacobian matrix $\frac{\partial \mathbf{g}}{\partial \tilde{\mathbf{x}}_{WI}}$ at the end of preintegration t_{k+1} , based on the zero initial condition (i.e., $\mathbf{P}_{m,0} = \frac{\partial \mathbf{g}_0}{\partial \tilde{\mathbf{x}}_{WI}} = \mathbf{0}_3$).

C. Odometry Measurement wrt. Extrinsic

1) *Spatial calibration*: Note that the preintegrated wheel measurement (16) provides *only* the 2D relative motion on the odometer's plane, while the VIWO state vector (17) contains the 6DOF IMU clone poses. We can define the following relative transformation between IMU clone states and leverage the extrinsic odometry/IMU calibration as:

$$\begin{bmatrix} {}^{O_k}\theta \\ {}^{O_k}\mathbf{d}_{O_{k+1}} \end{bmatrix} = \begin{bmatrix} \mathbf{e}_3^{\top} \text{Log}({}^I \mathbf{R}_G^{I_{k+1}} \mathbf{R}_G^I \mathbf{R}_I^{\top} \mathbf{R}^{\top}) \\ \Lambda ({}^I \mathbf{R}_G^I \mathbf{R}_G^I ({}^G \mathbf{p}_{I_{k+1}} + {}^{I_{k+1}} \mathbf{R}^{\top} \mathbf{p}_O - {}^G \mathbf{p}_{I_k}) + {}^O \mathbf{p}_I) \end{bmatrix} \quad (26)$$

where $\Lambda = [\mathbf{e}_1 \ \mathbf{e}_2]^{\top}$, \mathbf{e}_i is the i -th standard unit basis vector, and $\text{Log}(\cdot)$ is the $SO(3)$ matrix logarithm function [43]. As this measurement depends on the two consecutive poses as well as the odometer/IMU extrinsics, when updating with it in the MSCKF, the corresponding measurement Jacobians are needed and computed as follows [see (26) and (18)]:

$$\frac{\partial \mathbf{h}}{\partial \tilde{\mathbf{x}}_{I_{k+1}}} = \begin{bmatrix} \mathbf{e}_3^{\top} {}^I \hat{\mathbf{R}} & \mathbf{0}_{1 \times 3} & \mathbf{0}_{1 \times 9} \\ -\Lambda {}^I \hat{\mathbf{R}}_{I_{k+1}} \hat{\mathbf{R}}_{I_{k+1}} [{}^I \hat{\mathbf{p}}_O] & \Lambda {}^I \hat{\mathbf{R}}_G^I \hat{\mathbf{R}} & \mathbf{0}_{2 \times 9} \end{bmatrix} \quad (27)$$

$$\frac{\partial \mathbf{h}}{\partial \tilde{\mathbf{x}}_{O_{k+1}}} = \begin{bmatrix} -\mathbf{e}_3^{\top} {}^I \hat{\mathbf{R}}_{I_{k+1}} \hat{\mathbf{R}}_{I_{k+1}} & \mathbf{0}_{1 \times 3} \\ \Lambda {}^I \hat{\mathbf{R}}_{I_{k+1}} [{}^I \hat{\mathbf{p}}_{O_{k+1}} - {}^I \hat{\mathbf{p}}_{I_k}] & -\Lambda {}^I \hat{\mathbf{R}}_G^I \hat{\mathbf{R}} \end{bmatrix} \quad (28)$$

$$\frac{\partial \mathbf{h}}{\partial \tilde{\mathbf{x}}_{WE}} = \begin{bmatrix} \mathbf{e}_3^{\top} (\mathbf{I} - {}^{O_{k+1}} \hat{\mathbf{R}}) & \mathbf{0}_{1 \times 3} \\ \Lambda ({}^I \hat{\mathbf{R}}_{I_k} [{}^I \hat{\mathbf{p}}_{O_{k+1}}] + {}^{O_{k+1}} \hat{\mathbf{R}} [{}^O \hat{\mathbf{p}}_I]) & \Lambda (\mathbf{I} - {}^{O_{k+1}} \hat{\mathbf{R}}) \end{bmatrix} \quad (29)$$

where $[\cdot]$ is the skew symmetric matrix.

2) *Temporal calibration*: To account for the difference between sensor clocks and measurement delay, we model an unknown constant time offset between the IMU clock and the odometer clock:² $t_k = O_{t_k} + O_{t_I}$, where t_k and O_{t_k} are the times when measurement \mathbf{z}_k was collected in the IMU and odometer's clocks, and O_{t_I} is the time offset between the two time references. We wish to define the preintegrated odometry constraints between two clone poses which occur at the true IMU times t_k and t_{k+1} . Using the current best estimate of the time offset O_{t_I} , we can integrate our wheel encoder measurements between the odometer times $O_{t_k} =$

²We assume that the two wheel encoders are hardware synchronized and thus their readings have the same timestamps.

Algorithm 1 Odometry Measurement Update in VIWO

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1: procedure WHEEL_UPDATE( $\mathbf{x}_{k+1|k}, \{\omega_{ml}, \omega_{mr}\}_{k:k+1}$ )
2:   // Preintegrate measurements and Jacobian
3:    $\mathbf{z} = \mathbf{0}_{3 \times 1}, \frac{\partial \mathbf{g}}{\partial \tilde{\mathbf{x}}_{WI}} = \mathbf{P}_m = \mathbf{0}_3$ 
4:   for  $\omega_{ml(\tau)}, \omega_{mr(\tau)} \in \{\omega_{ml(k:k+1)}, \omega_{mr(k:k+1)}\}$  do
5:      $\mathbf{z} = \mathbf{z} + \Delta \mathbf{z}$ 
6:      $\mathbf{P}_m = \Phi_{tr,\tau} \mathbf{P}_m \Phi_{tr,\tau}^{\top} + \Phi_{n,\tau} \mathbf{Q}_{\tau} \Phi_{n,\tau}^{\top}$ 
7:      $\frac{\partial \mathbf{g}}{\partial \tilde{\mathbf{x}}_{WI}} = \Phi_{tr,\tau} \frac{\partial \mathbf{g}}{\partial \tilde{\mathbf{x}}_{WI}} + \Phi_{WI,\tau}$ 
8:   end for
9:   // Compute residual and Jacobian
10:   $\tilde{\mathbf{z}} = \mathbf{z} - \mathbf{h}(\hat{\mathbf{x}}_I, \hat{\mathbf{x}}_C, \hat{\mathbf{x}}_{WE}, O_{t_I}, \hat{\mathbf{x}}_{WI})$ 
11:   $\mathbf{H} = \left[ \frac{\partial \mathbf{h}}{\partial \tilde{\mathbf{x}}_I} \ \frac{\partial \mathbf{h}}{\partial \tilde{\mathbf{x}}_C} \ \frac{\partial \mathbf{h}}{\partial \tilde{\mathbf{x}}_{WE}} \ -\frac{\partial \mathbf{g}}{\partial \tilde{\mathbf{x}}_{WI}} \ \frac{\partial \mathbf{h}}{\partial O_{t_I}} \right]$ 
12:  // Perform  $\chi^2$  test & update
13:  if  $\chi^2(\tilde{\mathbf{z}}, \mathbf{H}, \mathbf{P}_m) == Pass$  then
14:     $\hat{\mathbf{x}}_{k+1|k+1} = \text{EKF\_Update}(\hat{\mathbf{x}}_{k+1|k}, \tilde{\mathbf{z}}, \mathbf{H}, \mathbf{P}_m)$ 
15:  end if
16: end procedure

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$I_{t_k} - O_{t_I}$ and $O_{t_{k+1}} = I_{t_{k+1}} - O_{t_I}$, whose corresponding times in the IMU clock are:

$$I_{t'_k} := I_{t_k} - O_{t_I} + O_{t_I} = I_{t_k} + O_{t_I} \quad (30)$$

$$I_{t'_{k+1}} := I_{t_{k+1}} - O_{t_I} + O_{t_I} = I_{t_{k+1}} + O_{t_I} \quad (31)$$

After preintegration we have the 2D relative pose measurement between the times $I_{t'_k}$ and $I_{t'_{k+1}}$ while the corresponding poses in our state vector are at the times I_{t_k} and $I_{t_{k+1}}$. We employ the following first-order approximation to account for this small time-offset error:

$${}^G \mathbf{R}^{(I_{t'_k})} \approx (\mathbf{I} - [{}^{I(t_k)} \boldsymbol{\omega} O_{t_I}])_G^{(I(t_k))} \mathbf{R} \quad (32)$$

$${}^G \mathbf{p}_{I(t'_k)} \approx {}^G \mathbf{p}_{I(t_k)} + {}^G \mathbf{v}_{I(t_k)} O_{t_I} \quad (33)$$

Using this relationship, we can compute the measurement Jacobian of the time offset as follows [see (26)]:

$$\frac{\partial \mathbf{h}}{\partial O_{t_I}} = \begin{bmatrix} \mathbf{e}_3^{\top} {}^I \hat{\mathbf{R}} ({}^{I_{k+1}} \boldsymbol{\omega} - {}^{I_{k+1}} \hat{\mathbf{R}} {}^{I_k} \boldsymbol{\omega}) \\ \Lambda {}^I \hat{\mathbf{R}} ([{}^I \hat{\mathbf{p}}_{O_{k+1}}] {}^{I_k} \boldsymbol{\omega} - {}^{I_{k+1}} \hat{\mathbf{R}} [{}^I \hat{\mathbf{p}}_O] {}^{I_{k+1}} \boldsymbol{\omega} + {}^{I_k} \hat{\mathbf{v}}_{I_{k+1}}) \end{bmatrix} \quad (34)$$

Note that in our experiments we use the IMU angular rate measurement and the current estimate of velocity in computing the above Jacobian.

D. Odometry Measurement Update

At this point, we have obtained the preintegrated wheel odometry measurements along with their corresponding Jacobians which are readily used for the MSCKF update:

$$\begin{aligned} \tilde{\mathbf{z}}_{k+1} &:= \mathbf{g}(\omega_{ml(k:k+1)}, \omega_{mr(k:k+1)}, \hat{\mathbf{x}}_{WI}) - \mathbf{h}(\hat{\mathbf{x}}_I, \hat{\mathbf{x}}_C, \hat{\mathbf{x}}_{WE}, O_{t_I}) \\ &\approx \underbrace{\left[\frac{\partial \mathbf{h}}{\partial \tilde{\mathbf{x}}_I} \ \frac{\partial \mathbf{h}}{\partial \tilde{\mathbf{x}}_C} \ \frac{\partial \mathbf{h}}{\partial \tilde{\mathbf{x}}_{WE}} \ -\frac{\partial \mathbf{g}}{\partial \tilde{\mathbf{x}}_{WI}} \ \frac{\partial \mathbf{h}}{\partial O_{t_I}} \right]}_{\mathbf{H}_{k+1}} \tilde{\mathbf{x}}_{k+1} - \frac{\partial \mathbf{g}}{\partial \mathbf{n}_{\omega}} \mathbf{n}_{\omega} \end{aligned} \quad (35)$$

Note that similar to how we treat visual features, we also employ the Mahalanobis distance test to reject bad preintegrated odometry measurements (which can be due to some unmodelled errors such as slippage) and only those passing the χ^2 test will be used for EKF update. To summarize, the odometry measurement update are outlined in Algorithm 1.

V. OBSERVABILITY ANALYSIS

As system observability plays an important role for state estimation [32], [33], we perform the observability analysis to gain insights about the state/parameter identifiability for the proposed VIWO. For brevity, in the following we only present the key results of our analysis while the detailed analysis can be found in the companion technical report [42].

For the analysis purpose, in analogy to [44], we consider the following state vector which includes a single cloned pose of $\mathbf{x}_{I_{k-1}}$ and a single 3D point feature ${}^G\mathbf{p}_f$:

$$\mathbf{x}_k = [\mathbf{x}_{I_k}^\top \quad \mathbf{x}_{C_k}^\top \quad \mathbf{x}_{etc}^\top]^\top, \quad \mathbf{x}_{etc} = [\mathbf{x}_{WE}^\top \quad {}^O t_I \quad \mathbf{x}_{WI}^\top \quad {}^G \mathbf{p}_f^\top]^\top \quad (36)$$

The observability matrix for the linearized system is:

$$\mathbf{M} = [\mathbf{H}_0^\top \quad (\mathbf{H}_1 \Phi_{(1,0)})^\top \quad \dots \quad (\mathbf{H}_k \Phi_{(k,0)})^\top]^\top \quad (37)$$

where $\Phi_{(k,0)}$ is the state transition matrix which is not obvious when including the clone in the state vector and will be derived below, and \mathbf{H}_k is the measurement Jacobian at time step k (e.g., see (35)). If we can find matrix \mathbf{N} that satisfies $\mathbf{M}\mathbf{N} = \mathbf{0}$, the basis of \mathbf{N} indicate the unobservable directions of the linearized system.

A. State Transition Matrix

In the MSCKF-based linearized system, the state transition matrix corresponding to the cloned state essentially reveals the stochastic cloning process. To see this, first recall how the cloned states are processed [45]: (i) augment the state with the current IMU pose when a new image is available, (ii) propagate the cloned pose with zero dynamics, (iii) marginalize the oldest clone after update if reaching the maximum size of the sliding window. In the case of one clone, this cloning corresponds to the following operation:

$$\tilde{\mathbf{x}}_{k|k} \leftarrow \begin{bmatrix} \mathbf{I}_6 & \mathbf{0}_{6 \times 9} & \mathbf{0}_6 & \mathbf{0}_{6 \times 13} \\ \mathbf{0}_{9 \times 6} & \mathbf{I}_9 & \mathbf{0}_{9 \times 6} & \mathbf{0}_{9 \times 13} \\ \mathbf{I}_6 & \mathbf{0}_{6 \times 9} & \mathbf{0}_6 & \mathbf{0}_{6 \times 13} \\ \mathbf{0}_{13 \times 6} & \mathbf{0}_{13 \times 9} & \mathbf{0}_{13 \times 6} & \mathbf{I}_{13} \end{bmatrix} \tilde{\mathbf{x}}_{k|k} \quad (38)$$

where the third block row corresponds to the clone of the current pose. While one may model this state transition by including all the clones (with zero dynamics) as the initial state, we explicitly constrain the clone with a noise-free measurement constraint.

We now can unify the cloning and propagating of the current error state $\tilde{\mathbf{x}}_k$ in the following linear mapping:

$$\tilde{\mathbf{x}}_{k+1} = \underbrace{\begin{bmatrix} \Phi_{I_{11}(k+1,k)} & \Phi_{I_{12}(k+1,k)} & \mathbf{0}_6 & \mathbf{0}_{6 \times 13} \\ \Phi_{I_{21}(k+1,k)} & \Phi_{I_{22}(k+1,k)} & \mathbf{0}_{9 \times 6} & \mathbf{0}_{9 \times 13} \\ \mathbf{I}_6 & \mathbf{0}_{6 \times 9} & \mathbf{0}_6 & \mathbf{0}_{6 \times 13} \\ \mathbf{0}_{13 \times 6} & \mathbf{0}_{13 \times 9} & \mathbf{0}_{13 \times 6} & \mathbf{I}_{13} \end{bmatrix}}_{\Xi_{(k+1,k)}} \tilde{\mathbf{x}}_k \quad (39)$$

where $\Phi_I = \begin{bmatrix} \Phi_{I_{11}} & \Phi_{I_{12}} \\ \Phi_{I_{21}} & \Phi_{I_{22}} \end{bmatrix}$ is the IMU state transition matrix (see [44]). Note that \mathbf{I}_6 in the third block row copies the IMU pose of $\tilde{\mathbf{x}}_k$ into $\tilde{\mathbf{x}}_{k+1}$ as a cloned pose without changing its value, while the cloned state in $\tilde{\mathbf{x}}_k$ has been discarded (marginalized). The above operation clearly reveals the MSCKF cloning process and thus, we will leverage this linear system (39) for the ensuing analysis.

Specifically, during the time interval $[t_0, t_{k+1}]$, we have the following linear dynamic system:

$$\tilde{\mathbf{x}}_{k+1} = \underbrace{\Xi(t_{k+1}, t_k) \Xi(t_k, t_{k-1}) \dots \Xi(t_1, t_0)}_{\Xi(t_{k+1}, t_0)} \tilde{\mathbf{x}}_0 \quad (40)$$

$$\Xi_{(k+1,0)} = \begin{bmatrix} \Phi_{I_{11}(k+1,0)} & \Phi_{I_{12}(k+1,0)} & \mathbf{0}_6 & \mathbf{0}_{6 \times 13} \\ \Phi_{I_{21}(k+1,0)} & \Phi_{I_{22}(k+1,0)} & \mathbf{0}_{9 \times 6} & \mathbf{0}_{9 \times 13} \\ \Psi^{(k+1,0)} & \mathbf{0}_{6 \times 9} & \mathbf{0}_6 & \mathbf{0}_{6 \times 13} \\ \mathbf{0}_{13 \times 6} & \mathbf{0}_{13 \times 9} & \mathbf{0}_{13 \times 6} & \mathbf{I}_{13} \end{bmatrix} \quad (41)$$

We also enforce the constraint that the initial IMU pose and the clone state at time t_0 are identical, whose error states have the following constraint:

$$[\mathbf{I}_6 \quad \mathbf{0}_{6 \times 9}] \tilde{\mathbf{x}}_{I_0} = \tilde{\mathbf{x}}_{C_0} \quad (42)$$

With (40) and (42), we finally have the following state transition matrix $\Phi_{(k+1,0)}$ for our observability analysis:

$$\begin{bmatrix} \tilde{\mathbf{x}}_{I_{k+1}} \\ \tilde{\mathbf{x}}_{C_{k+1}} \\ \tilde{\mathbf{x}}_{etc} \end{bmatrix} = \underbrace{\begin{bmatrix} \Phi_{I(k+1,0)} & \mathbf{0}_{15 \times 6} & \mathbf{0}_{15 \times 13} \\ \mathbf{0}_{6 \times 9} & \Psi^{(k+1,0)} & \mathbf{0}_{6 \times 13} \\ \mathbf{0}_{13 \times 15} & \mathbf{0}_{13 \times 6} & \mathbf{I}_{13} \end{bmatrix}}_{\Phi_{(k+1,0)}} \begin{bmatrix} \tilde{\mathbf{x}}_{I_0} \\ \tilde{\mathbf{x}}_{C_0} \\ \tilde{\mathbf{x}}_{etc} \end{bmatrix} \quad (43)$$

B. Observability Properties

Based on the measurement Jacobians and state transition matrix [see (9), (35) and (43)], we are able to construct the observability matrix (37) of the MSCKF-based linearized system under consideration. Careful inspection of this matrix reveals the following results (see [42] for details):

Lemma 5.1: The proposed VIWO has the following observability properties:

- With general motions, there are four unobservable directions corresponding to the global position and the yaw angle as in VINS [44].
- As matrix blocks of the observability matrix that are related to the wheel-encoder's extrinsic and intrinsic calibration parameters highly depend on dynamic states and encoder's readings, these blocks can be full-rank given sufficient motions, implying that these parameters are observable, which has been validated in simulations in terms of convergence (see Fig. 3).
- We identify the following five degenerate motions that cause the odometer's calibration parameters to become unobservable, which might be commonly seen for ground vehicles:

Motion	Unobservable
Pure translation	${}^O \mathbf{p}_I, b$
1-axis rotation	${}^O \mathbf{p}_I$
Constant angular and linear velocity	${}^O t_I$
No left/right wheel velocity	r_l / r_r
No motion	${}^I \mathbf{R}, {}^O \mathbf{p}_I, r_l, r_r, b$

VI. SIMULATION VALIDATIONS

The proposed VIWO was implemented within the *Open-VINS* [46] framework which provides both simulation and evaluation utilities. We expanded the visual-inertial simulator to additionally simulate a differential drive robot which can only have velocity in the local x-direction (non-holonomic constraint [4]) and have listed the key simulation parameters in Table I. In order to validate the proposed calibration method, we tested VIWO on a large-scale neighborhood

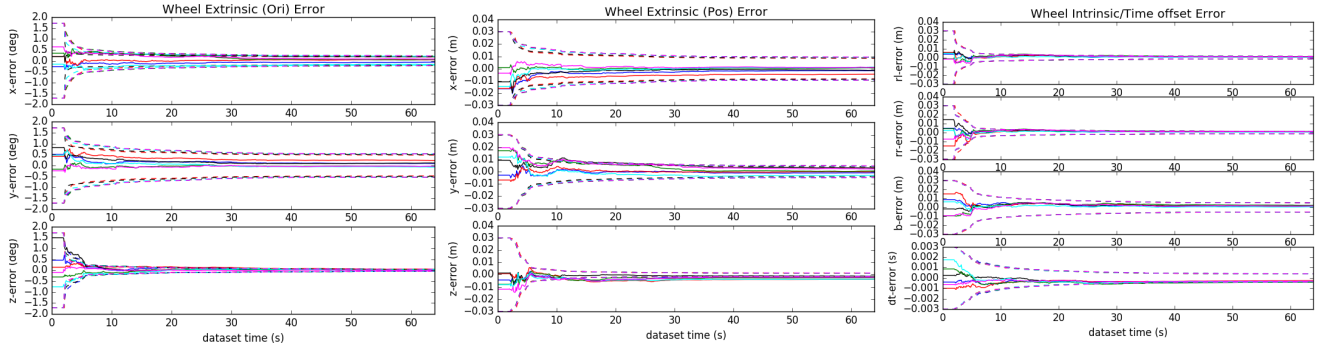


Fig. 3: Calibration errors of each parameter (solid) and 3σ bound (dotted) for six different runs under random motion. Each colors denote runs with different realization of the measurement noise and the initial values. Both the estimation errors and 3σ bounds can converge in about 10 seconds.

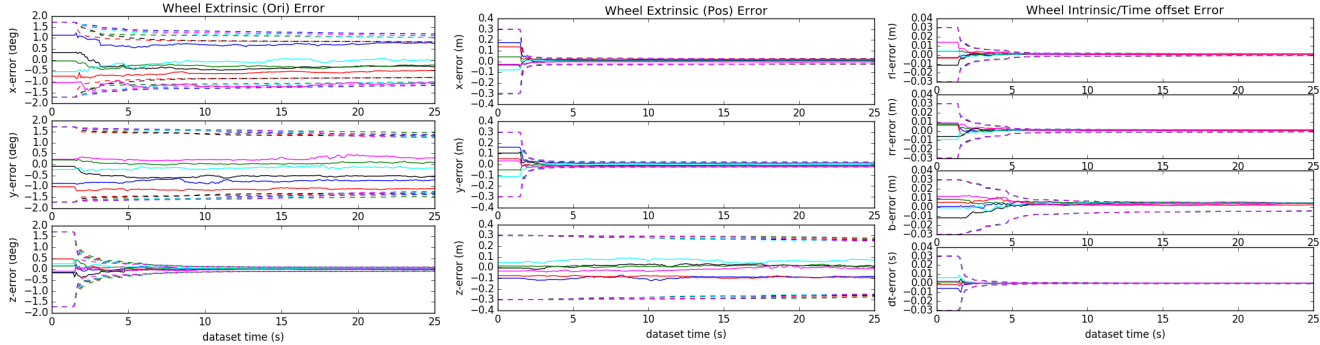


Fig. 4: Calibration error results of each parameter under planar motion. Each colors denote runs with different realization of the measurement noise and the initial values. Both the estimation errors (solid) and 3σ (dotted) bounds are reported.

TABLE I: Simulation parameters and prior single standard deviations that perturbations of measurements and initial states were drawn from.

Parameter	Value	Parameter	Value
Cam Freq. (hz)	10	IMU Freq. (hz)	200
Wheel Freq. (hz)	50	Num. Clones	15
Max. Feats	200	Feat. Rep.	GLOBAL
Pixel Proj. (px)	1	Wheel. White Noise	1.0e-03
Gyro. White Noise	1.0e-04	Gyro. Rand. Walk	1.0e-04
Accel. White Noise	1.0e-04	Accel. Rand. Walk	1.0e-04
Wheel Ext (Ori). Ptrb.	1.0e-02	Wheel Ext (Pos). Ptrb.	1.0e-01
Wheel Int. Ptrb.	1.0e-02	Wheel Toff. Ptrb.	1.0e-02

TABLE II: Relave pose error (RPE) of each algorithm (degree/meter).

	50m	100m	200m	NEES
VIO	0.362 / 1.252	0.494 / 2.245	0.657 / 3.930	3.921 / 3.895
true & w. cal.	0.277 / 0.550	0.365 / 0.908	0.479 / 1.573	1.952 / 2.020
true & wo. cal.	0.259 / 0.384	0.340 / 0.622	0.443 / 1.125	1.698 / 1.473
bad & w. cal.	0.276 / 0.543	0.365 / 0.888	0.486 / 1.526	1.943 / 1.826
bad & wo. cal.	0.572 / 0.510	1.104 / 1.142	2.239 / 3.367	59.678 / 183.538

simulation dataset with four different combinations: with “bad” or “true” initial calibration parameters and with or without online calibration. The bad initial values were drawn from the prior distributions specified in Table I. 50m, 100m, and 200m relative pose error (RPE, [47]) and the average normalized estimation error squared (NEES) results of the four configurations and standard VIO for comparison are shown in Table II. VIWO with “bad” initial values with calibration showed similar RPE results compare to those had “true” initial values, while VIWO with “bad” initial values without calibration has a very poor estimation performance and a large NEES due to treating the incorrect calibration as being true. Fig. 1 shows the trajectory of the algorithms, and clearly shows the failure of estimation when not performing calibration. To highlight the significance of the time offset,

TABLE III: RMSE of each algorithm with time offset 0.027 (degree/meter).

	true & wo. cal.	true & w. cal.	bad & w. cal.	bad & wo. cal.
RMSE	0.596 / 7.881	0.656 / 8.174	0.683 / 9.070	0.849 / 11.907
NEES	2.354 / 2.848	2.288 / 3.579	2.967 / 3.279	3.034 / 10.036

we also compared the algorithms with temporal calibration only. The time offset initial error was 0.027 seconds, while all other parameters were “true”. Table III reports the RMSE and NEES results and clearly shows that even small time offset errors can greatly degrade estimation performance.

A. Extrinsic, Intrinsic, and Time offset Calibration

To validate that all calibration parameters are able to converge for random motion, we simulated a trajectory which excited all-axes motions without invalidating non-holonomic constraint. Shown in Fig. 3, we performed online extrinsic, intrinsic, and time offset calibration six times with different initial perturbations. We additionally plot the 3σ bounds, which should bound the error in the case that our estimator is consistent. It is clear that all calibration parameters are able to quickly converge to near their true values and remain within 3σ bounds.

B. Degenerate Motion Validation

One of the degenerate motions that are of particular interest to grounded vehicles is planar motion. Fig. 4 shows the calibration results for the starting segment of the planar trajectory shown in Fig. 1. The extrinsic z-axis does not converge since the trajectory is in the global x-y plane and thus the robot only rotates around its z-axis causing this extrinsic to become unobservable. Note that the roll and pitch portion of the orientation (left top and left middle of Fig. 4) has a slower rate of convergence compared to other

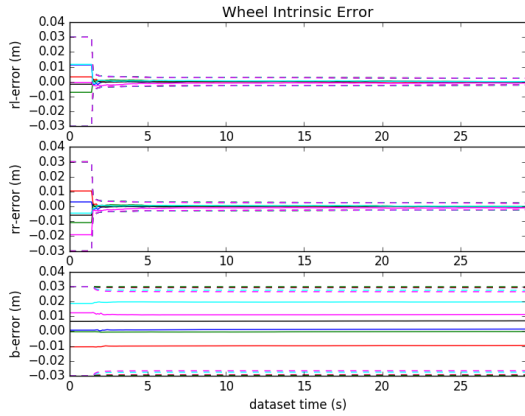


Fig. 5: Intrinsic calibration error under straight line motion

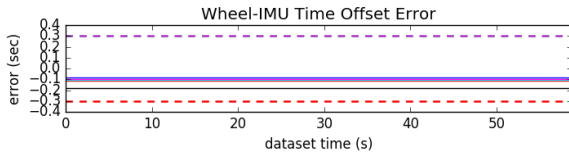


Fig. 6: Time offset calibration error under straight line motion

parameters because planar motion does not provide good geometrical constraints.

A straight line motion with constant linear velocity and no rotation, which is a special case of both pure translation and constant angular/linear velocities, is also simulated. To prevent the unobservable parameters from affecting other calibration, we only estimate either the wheel intrinsics or time offset and show their estimation results. Shown in Fig. 5, it is clear that the baselink is unable to converge, while the wheel radius can still converge due to having scale from the IMU. Fig. 6 shows the time offset is unable to be calibrated in this straight line/constant linear velocity case.

VII. REAL WORLD EXPERIMENTS

We further evaluate the proposed VIWO in a real-world dataset, KAIST *urban39* [9], which is collected in urban area with 11.06 km long trajectory, and used the stereo camera, IMU, and wheel encoder measurements for the estimation. We could not find any open sourced systems with the same sensor combination, thus compared VIWO with calibration (VIWO w. cal), VIWO without calibration (VIWO wo. cal), and VIO to evaluate the performance. Additionally, we also included the pure wheel odometry trajectory, to deliver a sense of wheel measurements errors. We used 25 clones and 200 features for the real-time estimation along with online IMU-camera intrinsic and spatiotemporal extrinsic calibration, and the resulting trajectories of each algorithm are shown in Fig. 7. The dataset provides the ground truth which is computed from graph SLAM. The root mean

TABLE IV: Values of calibration parameters before/after calibration

Parameter	Before	After
left wheel radius	0.311740	0.312262
right wheel radius	0.311403	0.311843
base length	1.52439	1.53201
Ext. Pos	[-0.070, 0.000, 1.400]	[-0.062, 0.003, 1.384]
Ext. Ori	[0.000, 0.000, 0.000]	[0.000, 0.001, -0.002]
Time offset	0.000	0.027

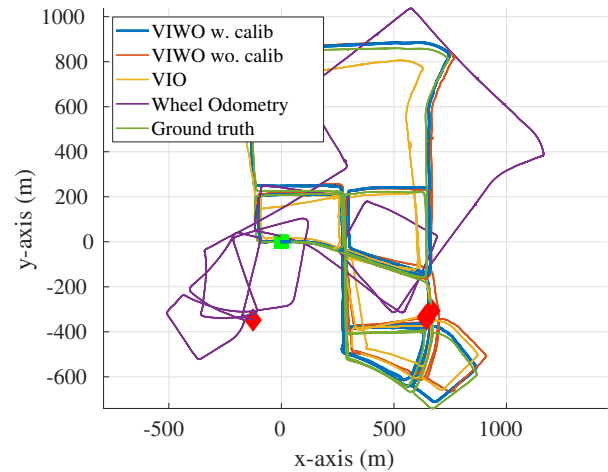


Fig. 7: *urban39* results of VIWO with calibration (blue), VIWO without calibration (red), VIO (yellow) and wheel odometry (purple). The green square and red diamond correspond to the start and end of the 11.06 km trajectory, respectively.

squared error (RMSE) of orientation and position of each algorithm were: VIWO w. cal 1.713 / 42.748, VIWO wo. cal 2.874 / 52.657, VIO 5.276 / 66.316, and Wheel Odometry 50.439 / 481.093 (degrees / meters). The initial values of parameters and the final calibrated values are listed in Table IV. We found the time offset calibration quickly converged to a nontrivial value of 0.027 seconds. As shown in simulation (see Table III), such small amount of time offset can deteriorate the estimation performance if not calibrated. The rest of the calibration parameters can also converge to values different from the initials, and this correction of calibration significantly improves the estimation accuracy.

VIII. CONCLUSIONS AND FUTURE WORK

In this paper, we have developed an efficient and consistent MSCKF-based visual-inertial-wheel odometry (VIWO) system that fuses multi-modal wheel-encoder, IMU and camera measurements. In particular, to compensate for possible time-varying calibration and for improving estimation, the proposed VIWO performs online sensor calibration of the spatiotemporal extrinsics of odometer-IMU/camera as well as the wheel encoder's intrinsics. To better understand the VIWO estimatability, we have conducted in-depth observability analysis for the MSCKF-based linearized system by leveraging a new state transition matrix that models the stochastic cloning in the MSCKF and have identified five degenerate motions that may cause online calibration unobservable. In the future, we will apply the observability analysis for dynamic estimator update.

REFERENCES

- [1] G. Huang, "Visual-inertial navigation: A concise review," in *Proc. International Conference on Robotics and Automation*, Montreal, Canada, May 2019.
- [2] K. J. Wu, C. X. Guo, G. Georgiou, and S. I. Roumeliotis, "Vins on wheels," in *2017 IEEE International Conference on Robotics and Automation (ICRA)*. IEEE, 2017, pp. 5155–5162.
- [3] X. Zuo, M. Zhang, Y. Chen, Y. Liu, G. Huang, and M. Li, "Visual-inertial localization for skid-steering robots with kinematic constraints," *arXiv preprint arXiv:1911.05787*, 2019.
- [4] R. Siegwart, I. R. Nourbakhsh, and D. Scaramuzza, *Introduction to autonomous mobile robots*. MIT press, 2011.

- [5] Y. Yang, P. Geneva, K. Eickenhoff, and G. Huang, "Degenerate motion analysis for aided INS with online spatial and temporal calibration," *IEEE Robotics and Automation Letters (RA-L)*, vol. 4, no. 2, pp. 2070–2077, 2019.
- [6] W. Lee, H. Cho, S. Hyeong, and W. Chung, "Practical modeling of gnss for autonomous vehicles in urban environments," *Sensors*, vol. 19, no. 19, p. 4236, 2019.
- [7] J. Borenstein and L. Feng, "Measurement and correction of systematic odometry errors in mobile robots," *IEEE Transactions on robotics and automation*, vol. 12, no. 6, pp. 869–880, 1996.
- [8] Y. Maddahi, N. Sepehri, A. Maddahi, and M. Abdolmohammadi, "Calibration of wheeled mobile robots with differential drive mechanisms: An experimental approach," *Robotica*, vol. 30, no. 6, pp. 1029–1039, 2012.
- [9] J. Jeong, Y. Cho, Y.-S. Shin, H. Roh, and A. Kim, "Complex urban dataset with multi-level sensors from highly diverse urban environments," *The International Journal of Robotics Research*, vol. 38, no. 6, pp. 642–657, 2019.
- [10] R. Kümmerle, G. Grisetti, and W. Burgard, "Simultaneous parameter calibration, localization, and mapping," *Advanced Robotics*, vol. 26, no. 17, pp. 2021–2041, 2012.
- [11] A. Martinelli, "Local decomposition and observability properties for automatic calibration in mobile robotics," in *2009 IEEE International Conference on Robotics and Automation*. IEEE, 2009, pp. 4182–4188.
- [12] N. Seegmiller, F. Rogers-Marcovitz, G. Miller, and A. Kelly, "Vehicle model identification by integrated prediction error minimization," *The International Journal of Robotics Research*, vol. 32, no. 8, pp. 912–931, 2013.
- [13] A. Kelly, "Fast and easy systematic and stochastic odometry calibration," in *2004 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)(IEEE Cat. No. 04CH37566)*, vol. 4. IEEE, 2004, pp. 3188–3194.
- [14] J. Deray, J. Solà, and J. Andrade-Cetto, "Joint on-manifold self-calibration of odometry model and sensor extrinsics using pre-integration," in *2019 European Conference on Mobile Robots (ECMR)*. IEEE, 2019, pp. 1–6.
- [15] J. L. Martinez, A. Mandow, J. Morales, S. Pedraza, and A. Garcia-Cerezo, "Approximating kinematics for tracked mobile robots," *The International Journal of Robotics Research*, vol. 24, no. 10, pp. 867–878, 2005.
- [16] G. Antonelli, F. Caccavale, F. Grossi, and A. Marino, "Simultaneous calibration of odometry and camera for a differential drive mobile robot," in *2010 IEEE International Conference on Robotics and Automation*. IEEE, 2010, pp. 5417–5422.
- [17] L. Heng, B. Li, and M. Pollefeys, "Camodocal: Automatic intrinsic and extrinsic calibration of a rig with multiple generic cameras and odometry," in *2013 IEEE/RSJ International Conference on Intelligent Robots and Systems*. IEEE, 2013, pp. 1793–1800.
- [18] D. A. Cucci and M. Matteucci, "Position tracking and sensors self-calibration in autonomous mobile robots by gauss-newton optimization," in *2014 IEEE International Conference on Robotics and Automation (ICRA)*. IEEE, 2014, pp. 1269–1275.
- [19] D. Li, K. Eickenhoff, K. Wu, Y. Wang, R. Xiong, and G. Huang, "Gyro-aided camera-odometer online calibration and localization," in *2017 American Control Conference (ACC)*. IEEE, 2017, pp. 3579–3586.
- [20] J. Liu, W. Gao, and Z. Hu, "Visual-inertial odometry tightly coupled with wheel encoder adopting robust initialization and online extrinsic calibration," in *2019 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*. IEEE, 2019, pp. 5391–5397.
- [21] X. Wang, H. Chen, Y. Li, and H. Huang, "Online extrinsic parameter calibration for robotic camera–encoder system," *IEEE Transactions on Industrial Informatics*, vol. 15, no. 8, pp. 4646–4655, 2019.
- [22] W. Lee, K. Eickenhoff, P. Geneva, and G. Huang, "Intermittent gps-aided vio: Online initialization and calibration," in *Proc. of the IEEE International Conference on Robotics and Automation*, Paris, France, 2020.
- [23] M. Li and A. I. Mourikis, "Online temporal calibration for camera–imu systems: Theory and algorithms," *The International Journal of Robotics Research*, vol. 33, no. 7, pp. 947–964, 2014.
- [24] T. Qin and S. Shen, "Online temporal calibration for monocular visual-inertial systems," in *2018 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*. IEEE, 2018, pp. 3662–3669.
- [25] K. Eickenhoff, P. Geneva, and G. Huang, "Sensor-failure-resilient multi-imu visual-inertial navigation," in *Proc. International Conference on Robotics and Automation*, Montreal, Canada, May 2019.
- [26] K. Eickenhoff, P. Geneva, J. Bloecker, and G. Huang, "Multi-camera visual-inertial navigation with online intrinsic and extrinsic calibration," in *Proc. International Conference on Robotics and Automation*, Montreal, Canada, May 2019.
- [27] Y. Yang, B. P. W. Babu, C. Chen, G. Huang, and L. Ren, "Analytic combined imu integration (*aci*²) for visual inertial navigation," in *Proc. of the IEEE International Conference on Robotics and Automation*, Paris, France, 2020.
- [28] F. Tungadi and L. Kleeman, "Time synchronisation and calibration of odometry and range sensors for high-speed mobile robot mapping," in *Proc. Australasian Conference on Robotics and Automation*, 2008.
- [29] R. Voges and B. Wagner, "Timestamp offset calibration for an imu-camera system under interval uncertainty," in *2018 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*. IEEE, 2018, pp. 377–384.
- [30] J. Rehder, R. Siegwart, and P. Furgale, "A general approach to spatiotemporal calibration in multisensor systems," *IEEE Transactions on Robotics*, vol. 32, no. 2, pp. 383–398, 2016.
- [31] P. Furgale, J. Rehder, and R. Siegwart, "Unified temporal and spatial calibration for multi-sensor systems," in *2013 IEEE/RSJ International Conference on Intelligent Robots and Systems*. IEEE, 2013, pp. 1280–1286.
- [32] G. Huang, "Improving the consistency of nonlinear estimators: Analysis, algorithms, and applications," Ph.D. dissertation, Department of Computer Science and Engineering, University of Minnesota, 2012.
- [33] J. A. Hesch, D. G. Kottas, S. L. Bowman, and S. I. Roumeliotis, "Observability-constrained vision-aided inertial navigation," *University of Minnesota, Dept. of Comp. Sci. & Eng., MARS Lab, Tech. Rep.*, vol. 1, p. 6, 2012.
- [34] Y. Yang and G. Huang, "Observability analysis of aided ins with heterogeneous features of points, lines and planes," *IEEE Transactions on Robotics*, vol. 35, no. 6, pp. 1399–1418, 2019.
- [35] G. Huang, A. I. Mourikis, and S. I. Roumeliotis, "Observability-based rules for designing consistent EKF SLAM estimators," *International Journal of Robotics Research*, vol. 29, no. 5, pp. 502–528, Apr. 2010.
- [36] A. Martinelli, "State estimation based on the concept of continuous symmetry and observability analysis: The case of calibration," *IEEE Transactions on Robotics*, vol. 27, no. 2, pp. 239–255, 2011.
- [37] A. Censi, A. Franchi, L. Marchionni, and G. Oriolo, "Simultaneous calibration of odometry and sensor parameters for mobile robots," *IEEE Transactions on Robotics*, vol. 29, no. 2, pp. 475–492, 2013.
- [38] A. I. Mourikis and S. I. Roumeliotis, "A multi-state constraint kalman filter for vision-aided inertial navigation," in *Proceedings 2007 IEEE International Conference on Robotics and Automation*. IEEE, 2007, pp. 3565–3572.
- [39] N. Trawny and S. I. Roumeliotis, "Indirect Kalman filter for 3D attitude estimation," University of Minnesota, Dept. of Comp. Sci. & Eng., Tech. Rep., Mar. 2005.
- [40] C. Hertzberg, R. Wagner, U. Frese, and L. Schröder, "Integrating generic sensor fusion algorithms with sound state representations through encapsulation of manifolds," *Information Fusion*, vol. 14, no. 1, pp. 57–77, 2013.
- [41] M. Li, "Visual-inertial odometry on resource-constrained systems," Ph.D. dissertation, UC Riverside, 2014.
- [42] W. Lee and G. Huang, "Visual inertial wheel odometry: Online calibration technical report," Robot Perception and Navigation Group (RPNG), University of Delaware, Tech. Rep., 2020. [Online]. Available: http://udel.edu/~ghuang/papers/tr_wheel-vio.pdf
- [43] G. Chirikjian, *Stochastic Models, Information Theory, and Lie Groups, Volume 2: Analytic Methods and Modern Applications*. Springer Science & Business Media, 2011, vol. 2.
- [44] J. A. Hesch, D. G. Kottas, S. L. Bowman, and S. I. Roumeliotis, "Consistency analysis and improvement of vision-aided inertial navigation," *IEEE Transactions on Robotics*, vol. 30, no. 1, pp. 158–176, Feb 2014.
- [45] Y. Yang and G. Huang, "Acoustic-inertial underwater navigation," in *Proc. of the IEEE International Conference on Robotics and Automation*, Singapore, May 29–Jun.3, 2017, pp. 4927–4933.
- [46] P. Geneva, K. Eickenhoff, W. Lee, Y. Yang, and G. Huang, "Openvins: A research platform for visual-inertial estimation," in *Proc. of the IEEE International Conference on Robotics and Automation*, Paris, France, 2020. [Online]. Available: https://github.com/rpng/open_vins
- [47] Z. Zhang and D. Scaramuzza, "A tutorial on quantitative trajectory evaluation for visual (-inertial) odometry," in *2018 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*. IEEE, 2018, pp. 7244–7251.